Robustness – theoretical framework

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Summary

More frequent use of advanced types of structures with limited redundancy and serious consequences in case of failure combined with increased requirements to efficiency in design and execution followed by increased risk of human errors has made the need of requirements to robustness of new structures essential. Further, the collapse of the World Trade Centre towers and a number of collapses of structural systems during the last 10 years has increased the interest in robustness. Typically modern structural design codes require that 'the consequence of damages to structures should not be disproportional to the causes of the damages'. However, despite the importance of robustness for structural design such requirements are not substantiated in more detail, nor have the engineering profession been able to agree on an interpretation of robustness which facilitates for its quantification. The ain of this fact sheet is to describe a theoretical and risk based framework to form the basis for quantification of robustness and for pre-normative guidelines.

Keywords

Robustness of structures, robustness indicators.

Background / Introduction

Robustness of structures has been recognized as a desirable property because of a several high system failures, such as the Ronan Point Building in 1968, where the consequences were deemed unacceptable relative to the initiating damage. After the collapse of the World Trade Centre, robustness has obtained a renewed interest, primarily because of the serious consequences related to failure of advanced types of structures and that consequences due to structural collapse may exceed the mere rebuilding costs by orders of magnitudes. Further, it was confirmed that robustness is strongly related to internal structural characteristics such as redundancy, ductility and joint behavior characteristics, but also that the consequences of structural collapse strongly depend on the specific scenario of events starting with some triggering event over a complex series of intermediate events involving more localized damages which finally lead to the collapse. In this scenario the extent to which consequences are generated depend not only on internal structural characteristics but may even more pronounced depend on passive and active measures for damage reduction as well as possible non-conformities with design assumptions due to the quality of execution and or maintenance.

In order to minimize the likelihood of failures as those mentioned above many modern building codes consider the need for robustness in structures and provide strategies and methods to obtain robustness. In fact, in all modern building codes, one can find a statement (in this or a slightly different form): “total damage resulting from an action should not be disproportional to the initial damage caused by this action”.

During the last decades there have been significant efforts to quantify aspects of robustness. When modelling robustness, system effects are very important. However, the primary criteria in building code are related to design and verification of sufficient reliability of components. It should also be noted that redundancy in systems is closely related to robustness. In principle redundant system are believed to be more robust than non-redundant systems – but this is not always the case as illustrated by the failures of the Ballerup Super Arena and the Bad Reichenhall Icehall, see (Früwald et al. 2007) and (Winter & Kreuzinger 2008).

In the Ballerup Super Arena collapse two out of 12 main trusses failed. The trusses in the primary load bearing system were made of glued laminated timber by a new, innovative design. A human error in design of the joints of the trusses was the main reason for the collapse. The transverse purlins (secondary system) were designed in such a way that progressive collapse of the whole roof should not occur in case of failure of a single main truss. The roof system can therefore be considered as a robust system in the sense that the whole roof did not collapse. This seems to be a good strategy in case of design/human errors occurring in many places / joints (high correlation) for new, unconventional structures.

In the Bad Reichenhall Icehall, the total roof collapsed progressively starting in one end of the arena. The primary structural system consists of very high box-girder beams with no previous experience. The secondary system was relatively stiff implying that the roof could be considered as a parallel system. Design, execution and operational errors in all main beams implied that the load bearing capacity was significantly lower than required and the roof collapsed with a snow load about ½ of the design snow load. The roof system can therefore not be considered as a robust system in the sense that the whole roof collapsed. It seems not to be a good strategy to use a parallel system when design/human errors occur in many places / beams (high correlation).

Methodology

Robustness is related to scenarios where exposures including unintentional and unforeseen loads and defects result in local damage to the structural system, and where this damage may lead to further collapse of the structure.

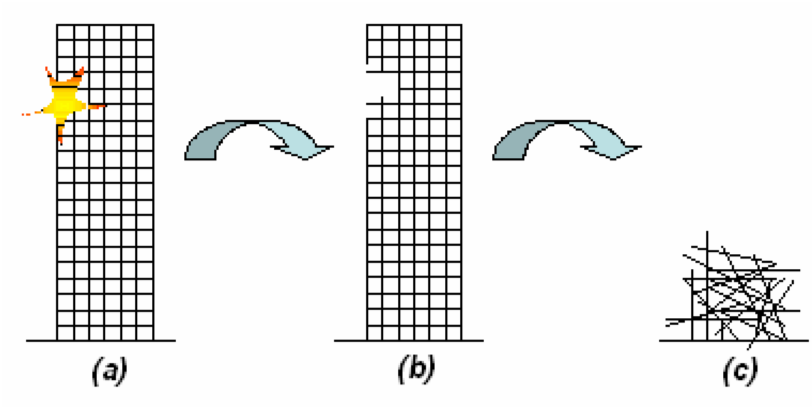

Figure 1: Illustration of the basic concepts in robustness (Eurocode EN 1991-1-7).

An illustration is presented in Figure 1 (from Eurocode EN 1991-1-7). Due to an exposure (a) of any kind, local damage (b) may occur. This local damage is defined as the direct consequence of the exposure. Given this local damage the structure may survive or (a substantial part) may collapse. Robustness requirements are especially related to step from b) to c), i.e. to avoid that a local damage develops to total collapse.

During the last decades there has been a significant effort to develop methods to assess robustness and to quantify aspects of robustness. The basic and most general approach is to use a risk analysis where both probabilities and consequences are taken into account. Approaches to define a robustness index can be divided in the following levels with decreasing complexity:

- A risk-based robustness index based on a complete risk analysis where the consequences are divided in direct and indirect risks
- A probabilistic robustness index based on probabilities of failure of the structural system for an undamaged structure and a damaged structure
- A deterministic robustness index based on structural measures, e.g. pushover load bearing capacity of an undamaged structure and a damaged structure

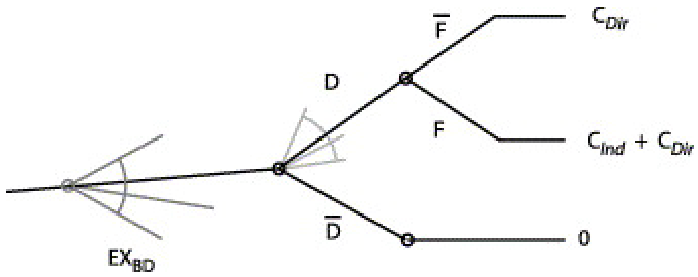

Figure 2: An event tree for robustness quantification, (Baker et al. 2008).

Figure 2 presents the same idea as in figure 1 in a more general way in the form of an event tree. The assessment starts with the consideration and modelling of exposures (*EX*) that can cause damage to the components of the structural system. The term “exposures” refers to extreme values of design loads, accidental loads and deterioration processes but also includes human errors in the design, execution and use of the structure. The term “damage” refers to reduced performance or failure of individual components of the structural system.

After the exposure event occurs, the components of the structural system either remain in an undamaged state ($\overline{D}$) as before or change to a damage state (*D*). Each damage state can then either lead to the failure of the structure (*F*) or no failure ($\overline{F}$).

Consequences are associated with each of the possible damage and failure scenarios, and are classified as either direct (C_{dir}) or indirect (C_{ind}). Direct consequences are considered to result from damage states of individual component(s). Indirect consequences are incurred due to loss of system functionality or failure and can be attributed to lack of robustness (Baker et al. 2008) and (JCSS 2008).

The basic framework for risk analysis is based on the following equation with risk contributions from local damages (direct consequences) and comprehensive damages (follow-up / indirect consequences), are added, see (Baker et al. 2008) and (JCSS 2008):

$$R = \sum_i \sum_j C_{\text{dir},ij} P\left(D_j \middle| E_i\right) P\left(EX_i\right) + \sum_k \sum_i \sum_j C_{\text{ind},ijk} P\left(S_k \middle| D_j \cap EX_i\right) P\left(D_j \middle| EX_i\right) P\left(EX_i\right) \tag{1}$$

where

$C_{\text{dir,ij}}$ consequence (cost) of damage (local failure) D_{j} due to exposure EX_{i}

$C_{\text{ind,ij}}$ consequence (cost) of comprehensive damages (follow-up / indirect) S_{k} given local damage D_{j} due to exposure EX_{i}

$P(EX_{\text{i}})$ probability of exposure EX_{i}

$P(D_{\text{j}}|EX_{\text{i}})$ probability of damage D_{j} given exposure EX_{i}

$P(S_{\text{k}}|...)$ probability of comprehensive damages S_{k} given local damage D_{j} due to exposure EX_{i}

The optimal design (decision) is the one minimizing the sum of costs of mitigating measures and the total risk *R*. A detailed description of the theoretical basis for risk analysis can be found in (JCSS 2008). It is noted that an important step in the risk analysis is to define the system and the system boundaries.

The total probability of comprehensive damages / collapse associated to (1) is:

$$P(\text{collapse}) = \sum_i \sum_j P\left(\text{collapse} \middle| D_j \cap EX_i\right) P\left(D_j \middle| EX_i\right) P\left(EX_i\right) \tag{2}$$

where $P\left(\text{collapse} \middle| D_j \cap EX_i\right)$ is the probability of collapse (comprehensive damage) given local damage D_j due to exposure EX_i. Note that compared to (1) only one comprehensive damage (collapse) is included in (2).

For damages related to key elements the probability of collapse is $P(\text{collapse} | D_j \cap EX_i) \approx 1$. From equation (2) it is obvious that the probability of collapse can be reduced by:

- Reducing one or more of the probabilities of exposures $P(EX_{\text{i}})$- prevention of exposure or event control

- Reducing one or more of the probabilities of damages $P(D_j|EX_i)$ - related to element/component behaviour
- Reducing one or more of the probabilities $P(\text{collapse}|D_j \cap EX_i)$

If the consequences are included in a risk analysis then also reduction of direct (local) consequences, $C_{dir,ij}$ and comprehensive (indirect) consequences, $C_{ind,ij}$ are important.

According to the description above and the robustness definition in (EN1990:2002), robustness is mainly related to the reduction of the probability $P(\text{collapse}|D_j \cap EX_j)$. Increasing the robustness at the design stage will in many cases only increase the cost of the structural system marginally – the key point is often to use a reasonable combination of a suitable structural system and materials with a ductile behaviour. In other cases increased robustness will influence the cost of the structural system.

Risk-based robustness index

(Baker et al. 2008) proposed a definition of a robustness index. The approach divides consequences into direct consequences associated with local component damage (that might be considered proportional to the initiating damage) and indirect consequences associated with subsequent system failure (that might be considered disproportional to the initiating damage). An index is formulated by comparing the risk associated with direct and indirect consequences. The index of robustness (I_{rob}) is defined as

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{3}$$

where R_{Dir} and R_{Ind} are the direct and indirect risks associated with the first and the second term in equation (1). The index takes values between zero and one, with larger values indicating larger robustness.

As mentioned above the optimal decision is the one which minimizes the total risk obtained by equation (1). This could equally well be by reducing the first or the second term in equation (1). This implies that the definition of a robustness index by equation (3) is not always fully consistent with a full risk analysis, but should be considered as a helpful indicator based on risk analysis principles. It is noted that since the direct risks typically are related to code based limit states they can generally be estimated with higher accuracy than the indirect risks.

Reliability-based robustness index

(Frangopol & Curley 1987) and (Fu & Frangopol 1990) proposed some probabilistic measures related to structural redundancy – which also indicates the level of robustness. A redundancy index (RI) is defined by:

$$RI = \frac{P_{f(\text{damaged})} - P_{f(\text{intact})}}{P_{f(\text{intact})}} \tag{4}$$

where $P_{f(\text{damaged})}$ is the probability of failure for a damaged structural system and $P_{f(\text{intact})}$ is the probability of failure of an intact structural system. The redundancy index provides a measure on the robustness / redundancy of the structural system. The index takes values between zero and infinity, with smaller values indicating larger robustness.

They also considered the following related redundancy factor:

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}} \tag{5}$$

where β_{intact} is the reliability index of the intact structural system and β_{damaged} is the reliability index of the damaged structural system. The index takes values between zero and infinity, with larger values indicating larger robustness.

Deterministic robustness index

A simple and practical measure of structural redundancy (and robustness) used in the offshore industry is based on the so-called *RIF* – value (Residual Influence Factor), (ISO19902 2007).

A Reserve Strength Ratio (*RSR*) is defined as:

$$RSR = \frac{R_c}{S_c} \tag{6}$$

where R_c denotes characteristic value of the base shear capacity of an offshore platform (typically a steel jacket) and S_c is the design load corresponding to ultimate collapse.

In order to measure the effect of full damage (or loss of functionality) of structural member no *i* on the structural capacity the so-called *RIF* –value (sometimes referred to as the Damaged Strength Ratio) is defined by:

$$RIF_i = \frac{RSR_{\text{fail},i}}{RSR_{\text{intact}}} \tag{7}$$

where RSR_{intact} is the *RIF*-value of the intact structure and $RSR_{\text{fail},i}$ is the *RIF*-value of the structure where member no *i* is failed/removed. The *RIF* takes values between zero and one, with larger values indicating larger robustness.

Other simple measures of robustness have been proposed based on e.g. the determinant of the stiffness matrix of structure with and without removal of elements.

Robustness in codes of practice

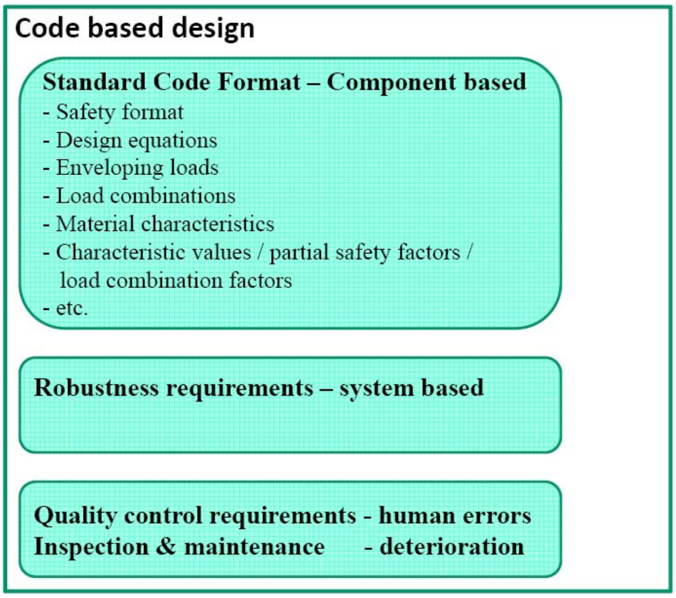

Figure 3: Code based design.

In many codes of practice as e.g. the Eurocodes the primary design requirements are related to checking that each component / element / connection has sufficient reliability. A sufficient reliability level is secured by using characteristic values and partial safety factors calibrated to a reliability level which typically correspond to an annual probability of failure of the order 10^{-6}. However, additional requirements / measures are needed to secure that the structure also as a system has sufficient reliability. Further, provisions are needed to reduce / eliminate the effect of design errors, execution errors, unexpected deterioration of components, etc. Robustness requirements in codes of practice should cover these aspects together with quality control systems and application of best practices in design, execution and operation & maintenance as illustrated in figure 3. It is noted that many codes of practice contain some robustness rules, e.g. requirements to tie together concrete elements, but the rules / provisions are not formulated in a consistent way on a rational basis. In countries where structures are designed for seismic loads the requirements to obtain earthquake resistant structures include many of the same aspects as those considered good for robustness, e.g. redundancy and ductility.

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