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Robustness of Structures
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Evaluation of Robustness of Structures – A Comparative Study

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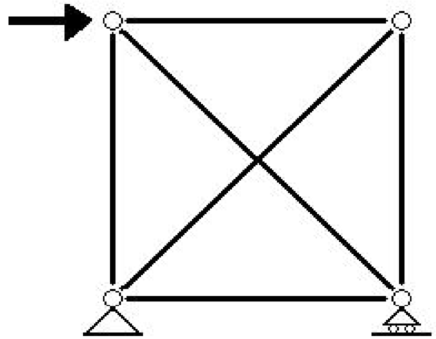
Objective of Study



- Investigation of different design approaches for a considered truss structure.

- Evaluation of robustness based on different indicators obtained from literature.

Methodology of Study

- Three design approaches considered:
  - **Code based design** using the Eurocode approach.
  - **System reliability based design** based on a requirement for system reliability.
  - **Risk based design** explicitly taking into account the consequences of failure.
- Design optimization for commercially available sections to minimize material volume carried out.

Methodology of Study

Evaluation of robustness carried out based on the following indicators:

- **Index of robustness**
  Baker, J.W., M. Schubert, and M.H. Faber (2008), "On the assessment of robustness, " *Structural Safety*, **30**(3), 253-267.

- **Vulnerability**
  Lind, N.C. (1995), "A measure of vulnerability and damage tolerance," *Reliability Engineering and System Safety*, **48**(1), 1–6.

- **Redundancy index**
  Fu, G. and D.M. Frangopol (1990), "Balancing weight, system reliability and redundancy in a multiobjective optimization framework," *Structural Safety*, **7**(2-4), pp. 165–175.

- **Code based design**

Object function: $$\arg\min_{\mathbf{a}}(C\ (\mathbf{a}))$$

where:

$$C\ (\mathbf{a}) = \sum_{i=1}^{n_m} l_i a_i c$$

n_m = number of members

l_i = length of member

a_i = cross sectional area of member

c = costs per unit volume of material

Design Optimization

- **System reliability based design**

Object function: $$\arg\min_{\mathbf{a}}(C\ (\mathbf{a}))$$

Constraint: $$p_f^{system} \leq 10^{-4}\ pa$$

- **Risk based design**

Object function: $\arg\min_{\mathbf{a}}(C_{LS}(\mathbf{a},T))$

where:

$$C_{LS}(\mathbf{a},T)=\sum_{i=1}^{n_m} l_i a_i c + E\left[C_F(\mathbf{a},T)\right]$$

$E\left[C_F(\mathbf{a},T)\right]$ = expected value of discounted costs due to failure of system as well as failure of individual members over the life time T of the structure

Modelling of Loads, Resistances and Properties

Variable	Distribution Type	Mean μ	Standard Deviation σ
Load R	Gumbel	$\mu_R = 24$ [KN]	$\sigma_R = 0.2\mu_L$ [KN]
Yielding stress σ_y	LogNormal	$\mu_y = 250$ [MPa]	$\sigma_y = 0.07\mu_y$ [MPa]
Young's Modulus E	Deterministic	200000 [Mpa]	–
Section properties thickness t width w	 Normal Normal	 $\mu_t = t_{nom}$ [mm] $\mu_w = w_{nom}$ + Uni(-1,1) [mm]	 $\sigma_t = 0.04\mu_t$ [mm] σ_w = Uni(0,1) [mm]
Length of outer element Length of brace element	Deterministic Deterministic	3 [m] 4.24 [m]	– –

Evaluation of Component Reliability

- Limit states for component reliability assessment based on yielding and buckling criteria.

- **Yielding**

$$M_y = \sigma_y A_i - R_i$$

- **Buckling**

$$M_B = \pi^2 E I_i / (l_i)^2 - R_i$$

where:

R_i = calculated axial load on member

I_i = moment of inertia of member

Evaluation of System Reliability and Consequences

- **System failure** defined as the formation of a mechanism in the structure.

- **Direct consequences** associated with failure of individual components represented by material and component replacement costs.

- **Indirect consequences** associated with failure of entire structure represented as a multiple of direct consequences.

Results from Design Optimization

	Optimal Section			
Approach	**Cross-sections of brace elements [all in mm]**	**Cross-sections of outer elements [all in mm]**	**Initial Cost (Euros)**	**System probability of failure**
Code based design	60 x 60 x 3	50 x 50 x 2	425	$1.25 \cdot 10^{-4}$
System reliability based design	70 x 70 x 3	50 x 50 x 2	465	$3.4 \cdot 10^{-5}$
Risk based design (for very high indirect consequences)	70 x 70 x 3	60 x 60 x 2	505	$1.2 \cdot 10^{-5}$

Index of robustness (I_{Rob})

Baker, J.W., M. Schubert, and M.H. Faber (2008), "On the assessment of robustness, " *Structural Safety*, **30**(3), 253-267.

$$I_{Rob} = \frac{Direct\ Risk}{Direct\ Risk + Indirect\ Risk}$$

Index of robustness (I_{Rob})

	Index of Robustness		
Ratio of Indirect to Direct Consequences	**Code based design** 60 x 60 x 3 50 x 50 x 2	**System reliability based design** 70 x 70 x 3 50 x 50 x 2	**Risk based design** 70 x 70 x 3 60 x 60 x 2
5	**0.796**	0.731	0.551
10	0.280	**0.617**	0.499
20	0.197	**0.460**	0.366
40	0.135	**0.305**	0.252
80	0.093	0.194	**0.202**
100	0.058	0.121	**0.143**

Vulnerability (V)

Lind, N.C. (1995), "A measure of vulnerability and damage tolerance," *Reliability Engineering and System Safety*, **48**(1), 1–6.

$$V = V(r_d, S) = P(r_d, S) / P(r_0, S)$$

where:

S	=	loading
$P(r, S)$	=	probability of system failure in state r for loading S
r_0	=	pristine state of system
r_d	=	damaged state of system

Vulnerability (V)

- Dominant damage states considered:
  - Buckling failure of outer element
  - Buckling failure of brace element

- Probabilities of system failure given occurrence of damage states weighted with probabilities of damage states to obtain vulnerability.

	Vulnerability		
	Code based design 60 x 60 x 3 50 x 50 x 2	**System reliability based design** 70 x 70 x 3 50 x 50 x 2	**Risk based design** 70 x 70 x 3 60 x 60 x 2
Vulnerability	3.572	**2.947**	3.459

Redundancy index (RI)

Fu, G. and D.M. Frangopol (1990), "Balancing weight, system reliability and redundancy in a multiobjective optimization framework," *Structural Safety*, **7**(2-4), pp. 165–175.

$$RI = \frac{P_{f(dmg)} - P_{f(sys)}}{P_{f(sys)}}$$

where:

$P_{f(dmg)}$ = probability of component failure

$P_{f(sys)}$ = probability of system failure

Redundancy index (RI)

- Dominant damage states considered:
  - Buckling failure of outer element
  - Buckling failure of brace element

	Redundancy Index		
Damage state	**Code based design** 60 x 60 x 3 50 x 50 x 2	**System reliability based design** 70 x 70 x 3 50 x 50 x 2	**Risk based design** 70 x 70 x 3 60 x 60 x 2
Buckling failure of outer element	0.21	**3.29**	1.88
Buckling failure of brace element	0.96	1.79	**3.04**

Conclusion

- An evaluation of robustness based on different indicators obtained from literature has been presented.
- The vulnerability and redundancy measures consider robustness to be a measure concerning only the structure and depend on relative values of damage and failure probabilities.
- The study provides an indication of situations where consequences need to be considered in a risk based framework.