Concrete beams and slabs

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Summary

Concrete structures are usually designed based on small deformation theories. Every element is studied separately and only limited attention is given to the interaction between the different elements and their connections. In accidental situations large deformations will occur and the interaction between the different elements becomes important to provide alternate load paths.

For large deformations specialized material models are required. Using these large deformation models the geometrical non-linear behaviour of slabs and beams can be studied. In former years limiting cracks and deformations in concrete elements were one of the major design criteria. Therefore only limited experimental research is available to validate models describing large deformations.

In the current standards detailing rules are given. These reinforcements provide extra strength, ductility and continuity in constructions and should be taken into account during accidental situations. It is advised to implement the following aspects when designing constructions: load carrying capacity due to geometrical nonlinear behaviour, strength of connections between elements and the configuration of elements to provide alternate load paths.

To conclude an example is given of a two span beam subjected to large deformations. The arching and catenary action result in a much larger load carrying capacity than the design load, provided the edges are perfectly restrained.

Keywords

Robustness, arching action, catenary action, membrane action, redundancy, alternate load paths, geometrical nonlinear behaviour, reinforced concrete.

Background / Introduction

A number of structural failures in the second half of the 20th century have proven that adequate resistance of a structure is not obtained through the design of its individual elements only. The designer is therefore confronted with the fact that the connections of the different elements and the concept of the design have a significant influence on the resulting safety level of the structure and more importantly their sensitivity to progressive collapse.

This fact sheet discusses the use of concrete members in the design of structurally robust structures. Robustness is concerned with those events that are so unlikely to occur that traditional calculation methods only have a limited value. Therefore special attention is given to those situations where the geometrical and physical nonlinear behaviour dominates the load carrying capacity of concrete elements.

Problem statement / Key issues

In current engineering practice, the design of a concrete structure focuses on the design of its composing elements. Linear and quasi-linear design methods, based on the assumption of small deformations, are most commonly used to determine the size and strength of these elements. Only limited attention is given to the connections between elements. Usually the detailing of these connections is experience based or governed by the applied standards.

This element-by-element approach is quite pragmatic and doesn't take the structural integrity based on the global design of the structure into account. It also gives no implication of the behaviour of the structure under large deformations which are bound to occur when excessive damage is imposed on the structure during a calamity.

When designing concrete structures, especially those of consequence class 3, the following aspects should be included:

- the increased (or decreased) load carrying capacity due to geometrical nonlinear behaviour;
- the strength of the connections between elements, especially when large deformations are allowed in the design;
- the configuration of elements to provide alternate load paths to provide extra redundancy.

Material models

Current codes provide the designer with different models to describe the material behaviour of concrete and reinforcing steel. However, when large deformations or dynamic aspects of the analysis are taken into account different or adapted models are more suited to describe the material behaviour.

Time dependent phenomena like creep and shrinkage are beyond the scope of this document. Only the stress/strain relations of concrete and reinforcing steel and the bond-slip behaviour between these at ambient temperature are considered.

During the design of concrete elements in normal ULS and SLS the main strength parameters are obtained by applying the appropriate partial safety factors to the nominal values. The most commonly used models to describe the concrete behaviour only apply to those situations in which the deflections of the element are deemed small enough to be negligible (CEN, 2004). If nonlinear effects are taken into account these models don't suffice. To study the large deformation behaviour of concrete elements more realistic models should be applied which are based on the mean values of both concrete and reinforcing steel strength. This is especially true for those cases in which in-plane forces (arching, catenary or membrane action) contribute significantly to the overall stability of the structure. Using the design values usually leads to an overestimation of deflections and hence to an overestimation of the load carried by in-plane forces and the rotation capacity.

For a geometrical nonlinear analysis Eurocode 2 (CEN, 2004) and Model Code 1990 (CEB-FIP, 1991) propose a stress-strain relationship as defined in equation 1. When the strain is larger than the nominal ultimate strain ε_{cu1} the designer could either choose a zero stress, a curve tangent to the proposed curve in ε_{cu1} following an exponential path to zero or any other curve describing the behaviour of concrete at these large strains. For the tensile behaviour the concrete can be assumed linear elastic until f_{ctm} with exponential tail behaviour.

$$\frac{\sigma_c}{f_{cm}} = \frac{\left(k - \frac{\varepsilon_c}{\varepsilon_{c1}}\right) \cdot \frac{\varepsilon_c}{\varepsilon_{c1}}}{1 + (k-2) \cdot \frac{\varepsilon_c}{\varepsilon_{c1}}} \; ; k = 1.05 E_{cm} \cdot \frac{|\varepsilon_{c1}|}{f_{cm}} \quad where\ 0 \leq \varepsilon_c \leq \varepsilon_{cu1} \tag{1}$$

Only reinforcing steel with high ductility should be used when large deformations are allowed during the robustness design. A modified bilinear stress-strain relationship can be used to describe the material behaviour that includes hardening of the steel. For hot rolled, heat-treated bars the model in Figure 1 can be used, since it closely resembles the actual stress-strain relationship.

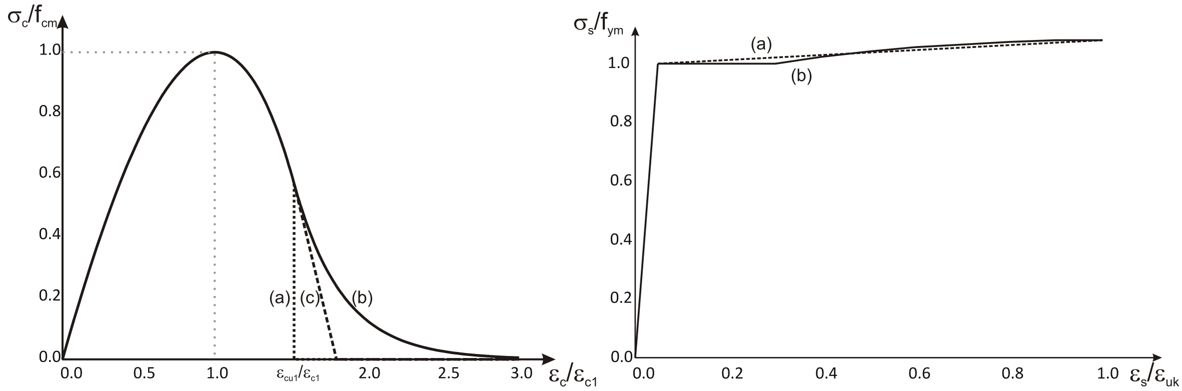

Figure 1: Left: Nonlinear stress-strain model for concrete in compression: (a) cut off, (b) exponential and (c) bilinear tail. Right: Nonlinear stress-strain model for reinforcing steel: (a) bilinear and (b) constant-parabolic

Model Code 1990 proposes a model to describe the bond-slip behaviour between reinforcing bars and concrete. At relatively small slip values the peak stress is reached which is followed by a bilinear softening branch. In large deformation analysis it can suffice to only model this last horizontal part of the bond-slip model to avoid negative pivots during finite element analysis.

The removal of a load bearing element usually occurs suddenly. In case of an explosion this is only a matter of a few milliseconds so that the structure responds dynamically to this event. Several authors have stated (Val and Val, 2006) that if the structure possesses sufficient ductility, the occurrence of large deformations and energy dissipation significantly reduces the impact factor. If the ratio of the maximum to the elastic deformation is for example 6 then the dynamic impact factor is already reduced to 1.1. Taking into account that the stiffness and strength of materials usually increases under impact loading, the dynamic effect may become negligible in most practical cases. Therefore a nonlinear static analysis should yield fairly accurate results if sufficient ductility is provided.

Redundancy in RC elements

The capacity of a structure to carry loads that exceed those that were specified during design depends on a wide variety of parameters. First of all the geometry of the elements can lead to a higher load carrying capacity. Also the ductility of the elements is often not taken into account during design. Other than that alternate load paths might be available in the structure so that failure of one element does not result in the overall failure of the structure.

Geometry

One of the major challenges in the case of excessive damage of a system is that the force distribution within the constituting elements can change drastically. Taking the case of a statically indeterminate beam with two equal spans as an example the moment at the central support will change sign when the central column fails (Figure 2). Most steel beams are prismatic and symmetrical, this section can resist a sagging moment of the same magnitude as the hogging moment it was originally designed for. Concrete elements however are – although mostly prismatic of geometry – not symmetrical in reinforcement. Therefore a failure of the central column would lead to immediate failure of the element if no precautions are taken to withstand the change in moment direction.

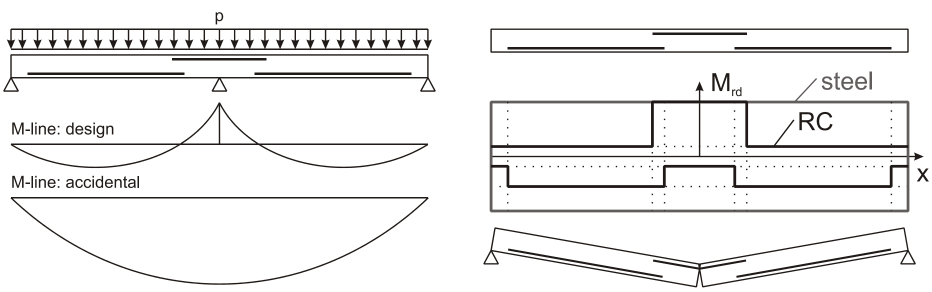

Figure 2: Lack of redundancy of RC beams in accidental situations

Concrete elements are usually sufficiently resistant to lateral instability problems. Only in the case of slender prestressed girders this problem might occur but only during the manipulation of the girder. Once the beam is in place the element has sufficient lateral support and is no longer susceptible to these lateral instability phenomena.

Ductility

The plastic material reserves of reinforced concrete depend on the difference between the design values and the actual mean values of concrete and reinforcing steel strength. For construction steel the JCSS probabilistic model code (JCSS, 2001) provides sufficient information to determine all necessary parameters to study the large deformation behaviour. Unfortunately no specific information is given to estimate the actual mean value of the ultimate tensile strength and ultimate strain of the reinforcing steel.

When linear elasticity is assumed in the analysis of an element there is also a linear relationship between the curvature and the bending moment in every section. As Figure 3 clearly shows the relationship between curvature and bending moment is nonlinear in nature. Only the first part of the graph can be considered linear. Once the critical moment is reached, and cracks form, the stiffness drops. The moment-curvature relationship in this branch is nonlinear due to tension stiffening. At the yield moment the reinforcing steel yields and the curvature rapidly increases until either the concrete crushes or the steel ruptures. The length of this last branch determines the ductility of the section subjected to bending.

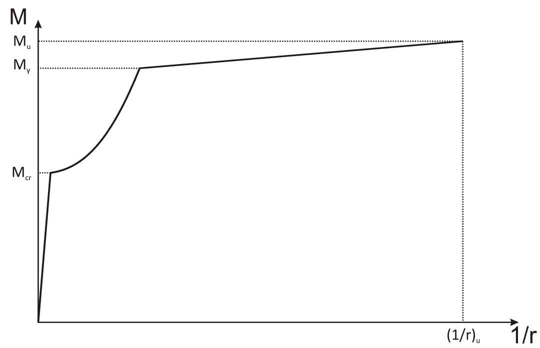

Figure 3: Lack of redundancy of RC beams in accidental situations

Force redistribution

Design methods that use the plasticity of the materials were first developed in steel construction. Steel is a very ductile material and therefore some economy can be obtained by utilizing the plastic reserves and force redistribution within steel structures. Things are slightly different in concrete elements. The amount of reinforcement is chosen based on the expected moment in that section. Therefore the use of plastic methods as an economy measure plays only a secondary role.

Plastic analysis does however allow for some freedom in the placement of reinforcing steel as shown in the following example (Figure 4). Considering the ultimate moment in the span is different from that at the central support, the virtual work theorem yields the ultimate load in equation 2. Assuming that the resisting moments are only depending on the tensile reinforcement in the considered section and the lever arm is constant the previous equation can be simplified (equation 2). This simplification proves that there is a certain freedom in the placement of reinforcing steel in statically indeterminate beams. As long as the ratio between the reinforcements is respected according to the simplified formula, the ultimate load of the system will be the same. This freedom is only limited by the rotation capacity of the sections and limitation of deflections and crack widths.

$$Q_u = M_{uv} \cdot \frac{l}{ab} + M_{us} \cdot \frac{1}{b} \approx \left(A_{sv} \cdot \frac{l}{ab} + A_{ss} \cdot \frac{1}{b} \right) f_y z \qquad (2)$$

Figure 4: Example beam for reinforcement redistribution in RC beams

In normal design procedures the acting moments are calculated based on a constant stiffness along the element. Due to cracks and the non-symmetrical placement of reinforcement the stiffness does not remain constant during loading. Regions with higher stiffness attract larger bending moments and a redistribution of moments is realized from the less stiff areas to the stiffer areas. For the previously mentioned beam the ratio of elastic moments is $m_{el} = M_s/M_v = 1.5385$. If the designer reinforces accordingly the moment redistribution after cracking leads to a moment ratio $m_g = 2.0427$. which differs significantly from the moment distribution as obtained by a linear theory.

Design based on nonlinear behaviour

Normal design methods as commonly used in the designer's practice are beyond the scope of this fact sheet. In what follows only those design methods are discussed which are particularly useful when damaged structures or structures near collapse are studied.

Yield line theory

The yield line theory is a method of designing reinforced concrete slabs. A yield line is a crack in a reinforced concrete slab across which the reinforcing bars are yielding. Due to this yielding a plastic rotation along this line occurs. The theory is based on the principle that the work done in rotating yield lines equals the work done due to load displacement. Since the method is based on the work method it will always overestimate the load carrying capacity of the plate. The aim is finding the yield line pattern that results in the minimal load capacity. The same principle can be used to design beams. In these linear elements plastic hinges are formed.

The basic yield line theory (Johansen) is based on small deformations. More advanced models have been developed (Bailey, 2001 and Bailey et al., 2008) to include in plane forces due to the large deflections and thermal loading.

Arching, catenary and membrane action

Slabs and beams are designed as flexural members i.e. they transfer loads by developing moment and shear stresses. At low deformations these assumptions are accurate, but at large deformations these members can also transfer loads by in plane stresses (or normal forces in the case of a beam).

A typical load-deflection curve of a slab with **full edge restraint** is shown in Figure 4 (Park, 1964). A similar curve is obtained when analysing beams as will be shown in the example. One can distinguish four phases:

- linear elastic phase (first part of AB);
- nonlinear phase with the developing of yield lines and compressive membrane forces (second part of AB);
- softening branch due to yielding of steel and disappearing of compressive membrane forces (part BC);
- hardening branch due to tensile membrane action of the reinforcement (part CD).

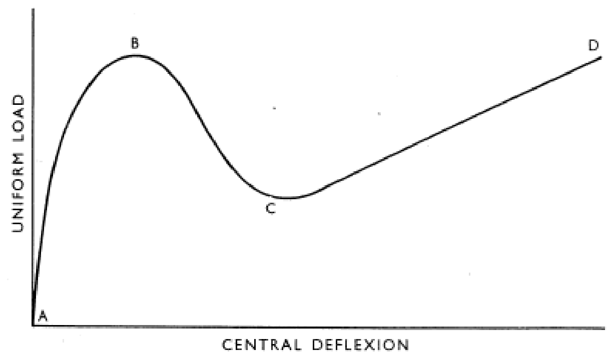

Figure 4: Schematic load-deflection curve of a slab with full edge restraint (Park, 1964)

It is observed that the ultimate load at B is significantly higher than the load calculated using Johansen's yield line theory. This is the result of compressive membrane forces that develop in the slab after cracking. As the deflection becomes larger the compressive membrane forces reduce to zero and full depth cracking of the slab occurs (point C). The membrane force becomes tensile and the slab's reinforcement grid acts as a plastic membrane (Park, 1964).

Perfectly unrestrained slabs are rare, but there are a lot of situations where the horizontal restraint is small or negligible, e.g. the corner panel of a continuous RC plate supported by columns.

Unlike horizontally restrained slabs, these slabs develop no compressive membrane action. However at large deflections tensile membrane action occurs in the central span region while a compressive ring develops around the slab's perimeter (Figure 5).

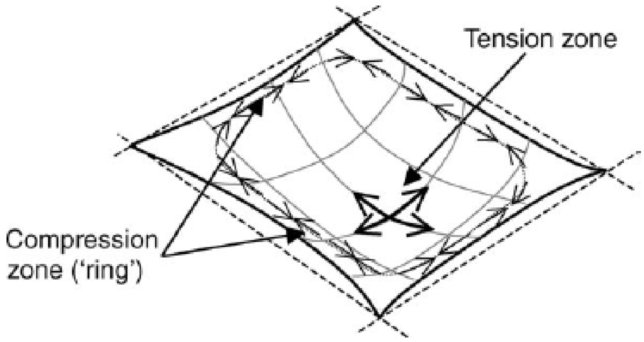

Figure 5: Membrane action of horizontally unrestrained slabs (Foster et al., 2004)

Rotation capacity

For these geometrical nonlinear effects as well as the earlier discussed moment redistribution to take place each section must have sufficient rotation capacity. It is the rotation of certain sections that create the necessary deformations to allow for phenomena like catenary action. Overestimating the rotation capacity can lead to a serious overestimation of catenary load carrying capacity.

Detailing rules based on current code provisions

Reinforcement requirements

In current design practice the main reinforcement in RC beams and slabs is calculated for the critical sections. Part of this reinforcement is curtailed based on the envelope of the acting tensile force. Design codes of RC structures also provide

- rules for minimum amounts of reinforcing steel e.g. to avoid brittle failures and excessive cracking;
- detailing rules e.g. reinforcement arrangements at intermediate and end supports of beams and slabs, in columns and walls, etc.;
- ductility conditions;
- reinforcement requirements for fire design.

It is useful to investigate to which extent these code provisions contribute to robustness in providing additional load paths or increasing the ultimate load. Traditionally, most RC structures have not explicitly been designed for robustness, and yet it can be observed that, in case of unexpected accidental loading, some degree of inherent robustness is available. This implies that the above mentioned rules could provide additional robustness without any additional costs.

Tying systems

In section 9.10 of EN 1992-1-1 (CEN, 2004) it is stated as a principle that concrete structures which are not explicitly designed to withstand accidental actions, shall have a suitable tying system to prevent progressive collapse by providing alternate load paths after local damage. The following types of ties have to be provided:

- peripheral ties (also along internal edges);
- internal ties, connected to the peripheral ties, in each floor and roof level in two directions at approximately perpendicular angles;
- horizontal ties at edge columns and walls;
- vertical ties in columns and/or walls in panel buildings of five storeys or more to limit the damage of collapse of a floor in the case of accidental loss of the column or wall below.

Limitations

Up until now, design methods have always focused on limiting cracks and deformations in concrete elements. Therefore only limited experimental research is available on the large deformation behaviour of concrete elements.

Because of this it is quite difficult to validate the results of finite element models subjected to large deformation behaviour, especially when simplified models are used. Detailed models of geometry and material properties have proven to be challenging in computation time, but especially in numerical stability.

Even when it is possible to predict the large deformation behaviour of a structural element, the designer needs to give special attention to the boundary conditions created by the rest of the structure. If the axial stiffness of the supports is overestimated, so will be the load carrying capacity due to arching or catenary action.

Not only the stiffness, but also the strength of the boundaries needs to be checked. The designer needs to be confident that the surrounding structure can withstand the axial and in plane forces that accompany catenary and membrane action. Pulling the surrounding columns inwards for example might result in buckling due to a larger imposed imperfection.

Example / Illustration / Case studies

A 3D finite element model was created to study the large deformation behaviour of a two span concrete beam (Decan and Taerwe, 2008). Two analyses were done, one including the central support and one without central support, simulating the loss of a column. The goal of the calculation was to check if the detailing rules in the Eurocode provide some extra robustness to RC beams. In this case horizontal tyings were provided and the bottom reinforcement continues over the central support.

Up until formation of the first cracks, the behaviour is linear elastic and no normal forces occur in the beam. When the first cracks appear the length of the beam tends to increase. Due to the axial constraints this leads to a compressive force (arching action) in the beam.

The load increases until plastic hinges form and the maximum load is reached. A softening branch follows as the compressive forces disappear. When the deflection continues to increase a tensile force develops in the longitudinal reinforcement. Due to catenary action these tensile forces can transfer loading well beyond the first load maximum until the ultimate tensile strain of the reinforcement bars is reached.

The load carrying capacity of the beam without central support seems to be very limited at small deflections. However at large deflections – usually larger than the height of the beam itself – the service load is reached.

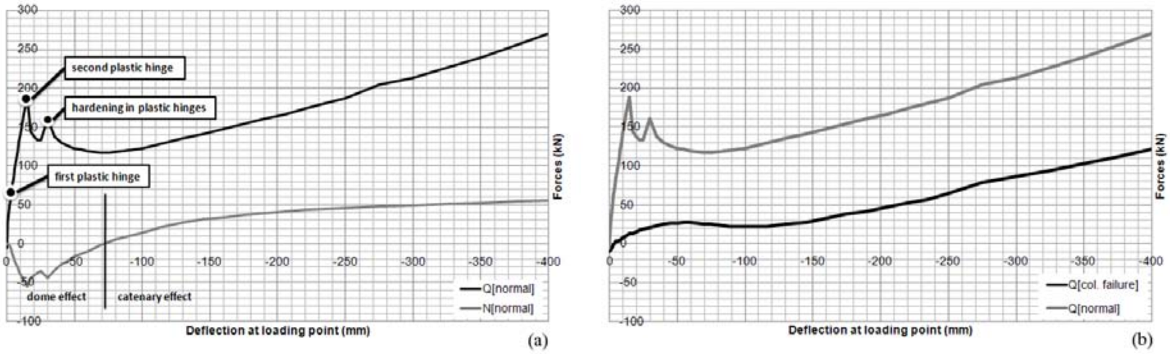

Figure 6: : Results of analysis of a two span beam – (a) applied load and normal force; (b) with and without central support

This example demonstrates that in the case of horizontal restraints a significant increase in load carrying capacity could be observed due to catenary action of the reinforcing steel at large deflections. In the unforeseen event of a column failure the beam would still be able to carry the design load, but with excessive deformation. The designer must verify if sufficient ductility/rotation capacity is available to withstand these deformations and if the connections are able to withstand the extra (normal) forces during catenary action.

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