# System reliability of timber structures – ductility and redundancy

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## Summary

For reduction of the risk of collapse in the event of loss of structural element(s), a structural engineer may take necessary steps to design a collapse-resistant structure that is insensitive to accidental circumstances e.g. by incorporating characteristics like redundancy, ties, ductility, key elements, alternate load path(s) etc. in the structural design. In general these characteristics can have a positive influence on system reliability of a structure however, in Eurocodes ductility is only awarded for concrete and steel structures but not for timber structures. It is well-know that structural systems can redistribute internal forces due to ductility of a connection, i.e. some additional loads can be carried by the structure. The same effect is also possible for reinforced concrete structures and structures of steel. However, for timber structures codes do not award that ductility will result in a semirigid behavior plus higher level of safety due to a lower probability that premature brittle failures occur and possible redistribution of forces for statically undetermined structures either internally in the joint or to other structural elements. A redistribution of forces, a so-called statical system effect, will usually increase the reliability of the whole structural system and give an extra safety margin compared to the deterministic code results. The aim of this fact sheet is to outline the relationship between system reliability and the characteristics ductility and redundancy.

## Keywords

System reliability, ductility, redundancy.

## Background / Introduction

In general when a structural system collapses one or more structural elements have failed. Such a failure mode can for any mechanical system be assigned to one of the following three categories: series systems, parallel systems or combination of series and parallel system (also referred as hybrid systems). In series systems failure of any element leads to the failure of the system. Parallel systems are those systems in which the combined failure of each and every element of the system results in the failure of the system [1]. Since a redistribution of the load effects takes place in a redundant structural system after failure of one or more of the structural elements it becomes very important in parallel systems to describe the behaviour of the failed structural elements after failure has taken place. If the structural element has no strength after failure the element is said to be *perfectly brittle*. If the element after failure has a load-bearing capacity equal to the load at failure, the element is said to be *perfectly ductile,* see figure 1. Clearly all kinds of structural elements and material behaviours cannot be described as perfectly brittle or perfectly ductile. All kinds of combinations in between exist, i.e. some, but not all, of the failure strength capacity is retained.

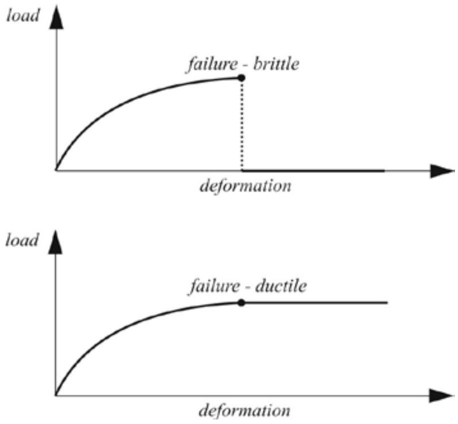


Figure1: Perfect brittle and prefect ductile failure mode behaviour

In the COST E 55 project - *Modeling of the Performance of Timber Structures* - existing numerical methods used to assess the reliability of timber structures are evaluated for their possible application to timber systems. Especially consensus on the general characteristics of timber systems regarding redundancy and robustness are established. To reach a better understanding of these aspects the following activities are considered within WG3:

- Characterisation of multi-scale variability in timber structures.
- Analysis of system effects for several types of timber structures.
- Qualification of robustness as a characteristic of timber structures.
- Establishing a framework for reliability based design and assessment of timber structural systems based on these considerations.

Related to these issues the relationship between robustness, system reliability and the characteristics ductility and redundancy are of great interest.

## System reliability – ductility and redundancy

For a structural system where the system reliability model is a series system of *m* failure elements a safety margin as a function of the basic variables **X** can be written

$$M_i = g_i(\mathbf{X}) \quad , \quad 1,2,\ldots,m \tag{1}$$

where $g_i$ is a limit state function. Then the probability failure of the system is given by

$$P_f^S = P\left(\bigcup_{i=1}^{m}\{M_i \le 0\}\right) = P\left(\bigcup_{i=1}^{m}\{g_i(\mathbf{X}) \le 0\}\right) \approx \Phi(-\beta^S) \tag{2}$$

where Φ is the standard normal distribution function and $\beta^s$ is the system reliability index for a series system. If a parallel system of *m* failure elements in one failure mode is considered then the probability of failure of the parallel system is defined as the intersection of the individual failure events

$$P_f^P = P\left(\bigcap_{i=1}^{m}\{M_i \le 0\}\right) = P\left(\bigcap_{i=1}^{m}\{g_i(\mathbf{X}) \le 0\}\right) \approx \Phi(-\beta^S) \tag{3}$$

As stated before it is very important for calculation of a parallel system reliability to describe the behaviour of the failed element after the failure has taken place. For the series system this is not very significant because when one element fails the failure of system is inevitable, i.e. a non-redundant system. However, before the reliability modelling in a parallel system of failure elements can be performed the structural behaviour of the considered failure mode must be clarified. More specifically the failure of the structural elements and consequences with determination of residual load-carrying capacity and load redistribution in each step in the structural element failure sequence must be described. Then the failure functions of the failure elements in the parallel system can be formulated. Failure function no. 1 models failure in parallel system element no. 1 without failure in any other elements. Failure function no. 2 models failure in parallel system element no. 2 with failure in the structural element corresponding to failure element no. 1 (i.e. after redistribution of loads). Failure function no. 3 then models failure of parallel system element no. 3 with failure in the structural elements corresponding to failure element nos. 2 and 1, etc. etc. If a stochastic load *S* is assumed and a parallel system consisting of *m* independently distributed element stochastic strengths $R_i$, see Figure 2, and a constant modulus of elasticity and perfect equal load sharing among the ideally brittle elements the system strength *R* can be calculated as

$$R = \max_{i=1}^{m}\{(m-i+1)\cdot R_i\} \tag{4}$$

where thr element strengths $R_i$ are set in a decreasing order, $R_1 < R_2 < \ldots < R_m$. Such a system is named a *Daniels system* and has be analysed in several papers with respect to system reliability for different assumptions related to stochastic variables [2-4].

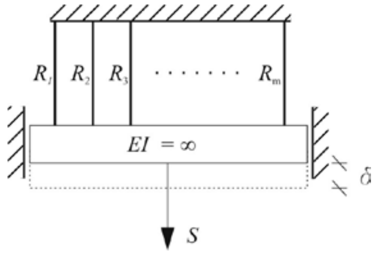


Figure 2: Mechanical model for parallel system

The corresponding system probability of failure is

$$P_f^P = P(R \le S) = P\left(\bigcap_{i=1}^{m}\left\{(m-i+1)\cdot R_i - S \le 0\right\}\right) \le \min_{i=1}^{m} P(\left\{(m-i+1)\cdot R_i - S \le 0\right\}) \qquad (5)$$

For an arbitrary stochastic force-deformation curve system failure occurs if the maximum system strength is exceeded by the load for a given imposed deformation δ, i.e. the probability of failure of the parallel system is given as the intersection of the individual failure events

$$P_f^P = \left(\bigcap_{\delta}\left\{R_i(\delta) - S \le 0\right\}\right) \qquad (6)$$

By using (4-6) [2-4] have presented results for probabilities of failure for the system in figure 2 under different post-failure member behaviors (ductility), correlations, strength and load variabilities and number of members. In general it is shown t that for a small number of elements the brittle system behaves much like the series system. As number of elements is increased the reliability of parallel system is increased significantly (and vice-versa for the series system). Further as the ductility increases linearly the reliability of the system increases much steeper (exponentially), so a relatively little ductility accounts for a considerable extra reliability. At last increases in correlation between elements imply a system reliability decrease. In summary, if there is a moderate degree of ductility, ductile systems will provide significant extra reliability only if elements are low correlated or with no correlation at all and if the load variability is not high. On the other hand, if there is a brittle behaviour, there is a relatively little effect of the system (especially for the small systems). There is even a small negative effect for medium coefficients of strength variation.

By using the model given by (4-6) relationship system reliability and redunancy have been investigated in several papers [4-8]. The terms redundancy, robustness and static indeterminacy are often used as synonymous. However, they can be interpreted as different properties of the structural system. Structural redundancy can be defined as the ability of the system to redistribute among its members the load which can no longer be sustained by some other damaged members. Structural robustness can instead be considered as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself. Often Redundancy is associated with the degree of static indeterminacy. However, in [5] it is demonstrated that the degree of static indeterminacy is not a consistent measure for structural redundancy. In fact, structures with lower degrees of static indeterminacy can have a greater redundancy than structures with higher degrees of static indeterminacy. It has been shown, that structural redundancy depends on many factors, such as structural topology, member sizes, material properties, applied loads and load sequence, among others [9]. For the analysis of redundancy [5] proposed some probabilistic measures related to structural redundancy – which also indicates the level of robustness. A redundancy index (*RI*) is defined by:

$$RI = \frac{P_{f(dmg)} - P_{f(sys)}}{P_{f(sys)}} \qquad (7)$$

where $P_{f\ (dmg)}$ is the probability of failure for a damaged structural system and $P_{f\ (sys)}$) is the probability of failure of an intact structural system. The redundancy index provides a measure on the robustness / redundancy of the structural system. They also considered the following related redundancy factor:

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}} \tag{8}$$

where $\beta_{\text{intact}}$ is the reliability index of the intact structural system and $\beta_{\text{damaged}}$ is the reliability index of the damaged structural system. It should a be noted that in general a redundant system is believed to be more robust than non-redundant systems – but this is not always the case as illustrated by the failures of the Ballerup Super Arena and the Bad Reichenhall icehall [10, 11]

## Example: System reliability – ductility analysis of timber structures

Related to the COST E55 project the effect of ductility in timber structures have been evaluated using the structural reliability framework in (4)-(6) [12]. The ductile behavior of joints as well as timber material in compression could have a positive influence on the robustness of timber structures [13-16]. During evolution trees have specialized in resisting their natural environment. In this respect it is a high quality fibre composite, optimally designed to resist loads acting on the tree but also to provide transport of water and nutritional agents. Stem and branches of the tree are designed to resist gravity loads and wind loads. The wood structure is adapted to create maximum strength in stressed directions, whereas in other directions the strength is quite low. As a result wood has special material properties like significant variability, anisotropy and orthotropic material properties consisting of “high strength” fibers (grains) oriented along the longitudinal axis of a timber log and packed together within a “low strength” matrix. Timber has no or a very little ductility in the tensile area, while in compressive linear elastic-plastic behaviour can be assumed [17]. In the aspect of timber joints all agree that the way to achieve high ductility is to take advantage of the plasticity of mechanical connectors (nails, dowels, bolts, etc.) The only certain way to create ductile structures is design in such a way that collapse of a structure is governed by failures of mechanical (ductile) joints [14]. This is especially important for the seismic behaviour of a timber structure. In the following investigations the level of ductility will be given as

$$D_f = \frac{\delta_f}{\delta_y} \tag{9}$$

where the yielding dicplacement $\delta_y$ and the ultimate displacement $\delta_f$ are defined in figure 3.

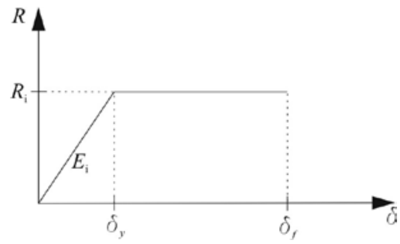


Figure 3: Force-deformation curve where element strength strength $R_i$ and modulus of elasticity (MOE) $E_i$ will be modelled as stochastic variables.

Another very important issue is that the joint ductility, elastic displacements, displacements at maximum load and ultimate displacements depend significantly upon the type of the connections used (dowel type fasteners, tooth plates and punched metal plates). There are also significant differences between different dowel type fasteners (bolts, dowels, nails, etc.). By using the definition in equation (9) a level of ductility in joints have been measured in the range 10–23 [14-16, 18]. From high grad timber material (C35 or C40) tests concerning deformation behaviour of a rectangular beam in bending a level of ductility in the range of 4-8 have been found [13, 16]. Based on these observations levels of ductility $D_f$ = 1, 2, 4, 8 have been studied related to the mechanical in figure 2. The system reliability versus number of elements for different levels of ductility $D_f$. is estimated based on Monte Carlo simulations where correlation between the strength of structural elements and load models for permanent and live load are introduced according to [19, 20]. For the present study robustness is defined as an increase in the ratio $\beta^s/\beta_e$, i.e. a relative increase in the system reliability $\beta^s$ compared to the element reliability index $\beta_e$. Each element in the mechanical system is assumed to have a one-year reliability index $\beta_e$ = 3.5 due to the permanent load when the load $S$ in the limit state function Equation (6) is modeled as

$$S = (1-\alpha)G + \alpha Q \tag{10}$$

$G$ is the permanent load and $Q$ is the variable load. $\alpha$ is a factor between 0 and 1, modeling the relative fraction of variable load which can be a imposed load or environmental load.□ Based on the tentative results in figure 4 it can be concluded that the system reliability of a structural timber system can be increased significantly awarding the ductile behaviour [12].

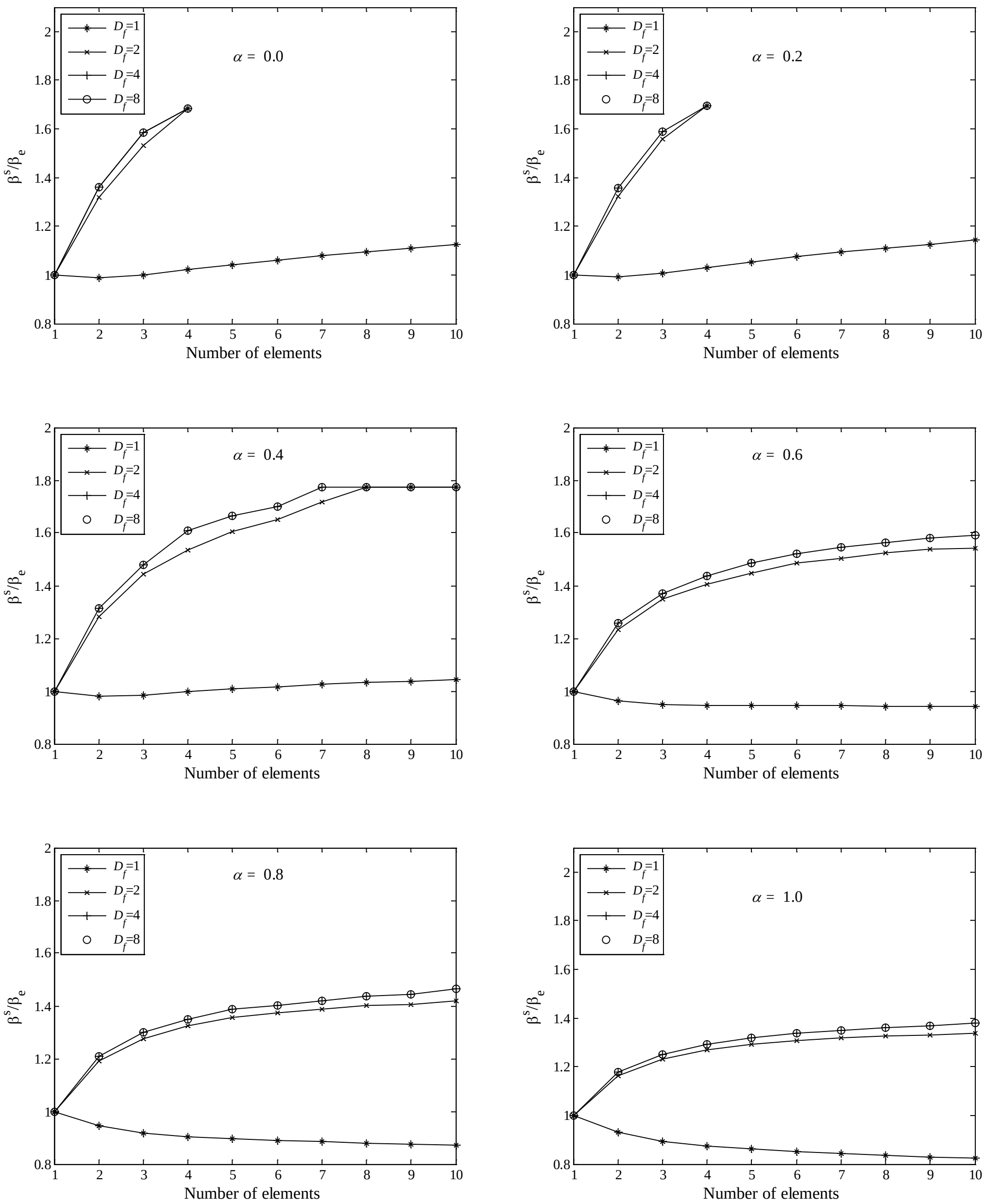


Figure 4: The ratio $\beta^s/\beta_e$ versus number of elements for different levels of ductility $D_f$ and the relative fraction of variable load $\alpha$.

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