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Building a Robustness Index

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Design Philosophy

- Optimal design: minimum cost & adequate performance.
- Reliability-oriented optimal design: the structural performance is usually judged based on reliability, which must be kept above a certain threshold.
- One-level optimization: instead of considering the reliability estimate as a self-standing optimization problem, it is included in the cost-benefit analysis by using the Kuhn and Tucker conditions (F_{KT}).
- Idea: also robustness can be introduced in the optimization problem as a further requirement for an adequate structural performance.

System Objectives

- Cost minimization → (F_C, w_C)
- Reliability requirement → $(-F_R, w_R)$
- Robustness → $(-F_I, w_I)$

where the weights w_X represent the Lagrange multipliers and increase with the safety margin within which the corresponding requirement is fulfilled.

Reliability & Robustness Oriented Optimal Design

Minimize:

Reliability Estimate
(Kuhn and Tucker Conditions)

$$F(\boldsymbol{x},\boldsymbol{u}_1,\ldots,\boldsymbol{u}_m)=w_C F_C(\boldsymbol{x})+w_R F_R(\boldsymbol{x})+w_{KT}F_{KT}(\boldsymbol{x},\boldsymbol{u}_1,\ldots,\boldsymbol{u}_m)+w_I F_I(\boldsymbol{x})$$

Cost - (Reliability Requirement) - (Robustness Requirement)

where:

$\boldsymbol{x}$ is the design parameters vector;

$\boldsymbol{u}_i$ is the $N\times 1$ vector of the transformed random variables from the original space to the standard normal space, $i=1,..,m$;

m is the number of failure modes.

Objective 1: Cost

from: Rackwitz, R. (2002). Optimization and Risk Acceptability Based on the Life Quality Index, *Structural Safety*, 24, 297-331.

- For a public structure whose reconstruction is systematic upon failure, which may only occur at the completion of the structure

$$F_C(\boldsymbol{x}) = C(\boldsymbol{x}) + [C(\boldsymbol{x}) + K]\left(\sum_{i=1}^{m} P_{f\,i}(\boldsymbol{x})\right)\bigg/\left(1 - \sum_{i=1}^{m} P_{f\,i}(\boldsymbol{x})\right)$$

where:

- $C(\boldsymbol{x})$ is the design and construction cost of the structure;
- $K = K_M + K_H$ is the sum of the direct cost of the structural failure (including both the damage and the debris removal), K_M, and the cost of saving human lives, K_H ;
- P_{fi} is the probability of the i-th failure mode.

Objective 2a: Reliability Requirement

- Imposing: $P_{fi} \leq 10^{-6}$

  for $i=1,\ldots,m$ (number of failure modes)

- Is equivalent to minimize:

$$F_R(\boldsymbol{x}) = -\sum_{i=1}^{m} \frac{10^{-6} - P_{fi}(\boldsymbol{x})}{P_{fi}(\boldsymbol{x})}$$

Objective 2b: Reliability Estimate

- Kuhn and Tucker Conditions:

$$F_{KT}(\boldsymbol{x},\boldsymbol{u}_1,\ldots,\boldsymbol{u}_m)=\sum_{i=1}^{m}\left[\,|\,g_i(\boldsymbol{u}_i,\boldsymbol{x})\,|+\sum_{j=1}^{N-1}\left|\left(u_{ij}\left\|\nabla\boldsymbol{g}_i(\boldsymbol{x})\right\|+\nabla g_{ij}\left\|\boldsymbol{u}_i(\boldsymbol{x})\right\|\right)\right|\,\right]$$

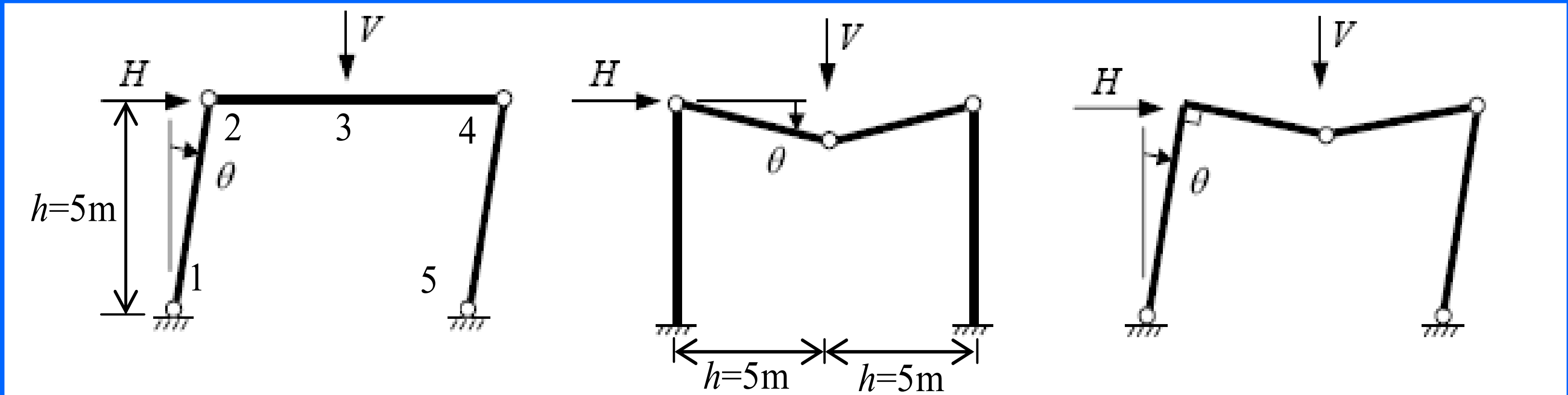
- where: $g_i(\boldsymbol{u}_i,\boldsymbol{x})$ is the i-th limit state function, with i=1,..,m.
- When, for any i, both the contributes to F_{KT} are null, $\|\boldsymbol{u}_i\|$ represents the value of the reliability index β_i for the i-th failure mode, from which one determines P_{fi} as Φ(- β_i) = Φ(-μ_{gi}/ σ_{gi}), being Φ(.) the standard normal cumulative distribution function.

Objective 3: Robustness

- Robust structures are those that develop the less catastrophic failure modes first → failure modes hierarchy

- Example: “weak beam/strong column” requirement in the design of a building

Portal-Frame Numerical Example



N = 9 Random Variables

Random Variable	Distribution	Mean	Standard Deviation
M_1, M_5	Normal	μ_a [kN m]	13.5 [kN m]
M_3	Normal	μ_b [kN m]	13.5 [kN m]
M_{2c}, M_{4c}	Normal	μ_a [kN m]	13.5 [kN m]
M_{2b}, M_{4b}	Normal	μ_b [kN m]	13.5 [kN m]
H	Normal	50 [kN]	15 [kN]
V	Normal	40 [kN]	15 [kN]

Robustness Index Formulation

- Robustness Function: $Z = M_{pc} - M_{pb} \geq 0$
- Corresponding Robustness Index:

$$I = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_c - \mu_b}{\sqrt{\sigma_c^2 + \sigma_b^2}} \geq 0$$

- (μ_b, σ_b), (μ_c, σ_c) = means and standard deviations of the plastic moments of the beams and of the columns, respectively.

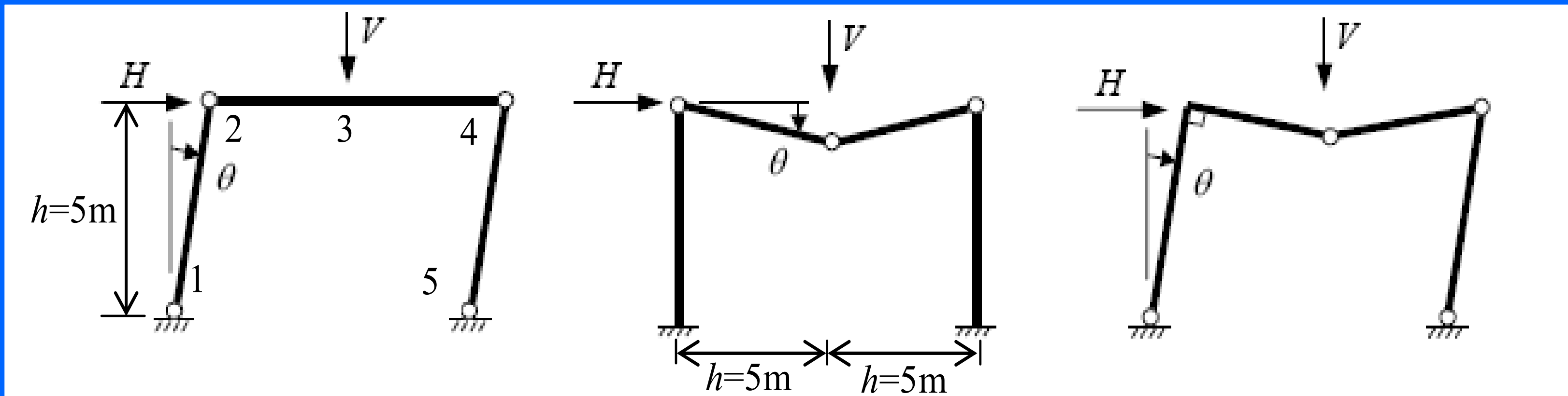
Optimization Problem Particularized for the Portal-Frame Example

- $\boldsymbol{x} = [\mu_c\ \mu_b]^T$ is the vector which collects the design parameters, so that :

$$F_I(\boldsymbol{x}) = -I = \frac{x_2 - x_1}{\sqrt{2} \cdot 13.5}$$

- $C(\boldsymbol{x}) = 3\mu_c + 2\mu_b$ is the design and construction cost;
- $K = K_M + K_H$, where: $K_M = 10^5$ is the direct cost of the structural failure,(including both the damage and the debris removal); $K_H = 2.4\ 10^6$ is the cost of saving human lives,
- Finally, $m = 3$ is the number of failure modes and P_{fi} is the probability of the i-th failure mode, computed by considering the following nonlinear limit state functions.

Nonlinear Limit State Functions



- Sway-mode: $g_1 = M_1 + \min(M_{2c}, M_{2b}) + \min(M_{4c}, M_{4b}) + M_5 - hH$
- Beam-mode: $g_2 = \min(M_{2c}, M_{2b}) + 2M_3 + \min(M_{4c}, M_{4b}) - hV$
- Complete mechanism: $g_3 = M_1 + 2M_3 + 2\min(M_{4c}, M_{4b}) + M_5 - hH - hV$

The corresponding gradients must be computed piecewise.

Solution via Differential Evolution (DE) Algorithm

<u>Advantages:</u>

- Search driven by objective function itself, instead of its gradient
- Independency of the results accuracy from the initial guess
- Few input parameters
- Moderate computational effort with respect to traditional Genetic Algorithms
- Easily adaptable to the solution of different problems

DE Solution Algorithm

Randomly generated initial population of NP vectors of the design parameters: $x \in S$ = search domain

NP = 160
Search range: (500, 1) kNm

x_i ($i = 1,\ldots, NP$) = a possible candidate to form the next generation

Mutant vector: $v_i = x_{r1} + \Gamma \cdot (x_{r2} - x_{r3})$

Γ = 0.8

Cross-over: $w_{ij} = v_{ij}$ if $rand_j \leq CR$, or
$w_{ij} = x_{ij}$ else

CR = 0.8

Selection: if $F(w_i) < F(x_i) \rightarrow w_i$
else $\rightarrow x_i$

Distance current optimal point from previous one > ε
Nr. Iterations < I

Y

N

End

$\varepsilon = 10^{-4}$
I = 5000

DE – Input Parameters

Initial Population Size, *NP*	160
Search Range - Upper Bound - Lower Bound	 500 kN m 1 kN m
Mutation Amplitude, Γ	0.5
Crossover Constant, *CR*	0.8
Tolerance for Convergence, ε	10^{-4}
Max Iterations Nr., I	5000

Objective Function Weights Calibration

$$F(\boldsymbol{x},\boldsymbol{u}_1,\ldots,\boldsymbol{u}_m)=w_C F_C(\boldsymbol{x})+w_R F_R(\boldsymbol{x})+w_{KT} F_{KT}(\boldsymbol{x},\boldsymbol{u}_1,\ldots,\boldsymbol{u}_m)+w_I F_I(\boldsymbol{x})$$

where the weights w_X represent the Lagrange multipliers and increase with the safety margin within which the corresponding requirement is fulfilled:

w_C	w_R	w_{KT}	w_I
:	6	1	70

Results: objectives evaluated at the optimal point

Case	μ_c	μ_b	I	Cost	Iterations
Without robustness	166.50	154.57	0.63	808.66	4212
With robustness	168.64	153.80	0.78	813.52	3098

Reliability Results

Failure mode	$g_i \times 10^{-5}$	β_i	$P_{fi} \times 10^{-5}$
Without robustness			
column	9.91	5.2190	0.0090
beam	23.84	5.3943	0.0034
complete	-10.73	4.5932	0.2183
With robustness			
column	15.4	5.1002	0.0170
beam	-0.5	5.0656	0.0204
complete	-55.43	4.6544	0.1624

Conclusions

- The robustness of a structure can be increased by including it in the design optimization problem. Of course, this happens with an increase of the costs.

- The desired hierarchy of failure modes can be achieved by properly adjusting the weights (Lagrangian multipliers) of the different terms in the objective function.

Future Developments

- Define extreme events scenarios and compute the conditional probabilities for each failure mode.
- Consider random failure in time by introducing out-crossing rates and interest rates.
- Generalize the process to different types of structural systems.