Department of Civil and Environmental Engineering
University of Cyprus
Nicosia – Cyprus

Design optimization methodologies to achieve structural robustness

Dimos C. Charmpis

Robustness of Structures
COST Action TU0601
1st Workshop, February 4-5, 2008, ETH Zurich, Zurich, Switzerland

Outline of the presentation

1. **Measuring structural robustness**
2. **Cost-oriented deterministic design optimization**
3. **RBDO**
   Reliability-Based Design Optimization
4. **RRBDO**
   Reliability and Robustness-Based Design Optimization
5. **MO-RRBDO**
   Multi-Objective Reliability and Robustness-Based Design Optimization
6. **Numerical example**
7. **RRBDO and MO-RRBDO as tools to compare measures and improvement strategies for robustness**
8. **Concluding remarks**

Measuring structural robustness – the Robustness Index *RI*

Baker, Schubert, Faber, *"On the assessment of robustness"*, Struct. Safety, 2007

Risk due to direct consequences

$$RI = \frac{R_{Dir}}{R_{Dir} + R_{Indir}}$$

Risk due to indirect consequences

A simple implementation

Damage probability ***Indirect consequences***

$$RI = \frac{P_d C_{Dir}}{P_d C_{Dir} + P_f C_{Indir}} = \frac{1}{1 + \frac{P_f}{P_d} C_r} \qquad C_r = \frac{C_{Indir}}{C_{Dir}}$$

Failure probability ***Direct consequences***

Cost-oriented deterministic design optimization

Design variable vector

Cost

$$\begin{array}{ll} \text{minimize} & C(\mathbf{d}) \\ \text{subject to} & g_j(\mathbf{d}) \geq 0,\ j=1,\ldots,n_g \\ & d_i \in D,\ i=1,\ldots,n_d \end{array}$$

Constraints
(e.g. satisfaction of Eurocodes provisions)

Design variable

Design space
(set of available design options)

- ***Topology***
- ***Shape***
- ***Sizing***

→ **Optimization**

RBDO – Reliability-Based Design Optimization

- **Single-objective optimization under uncertainty:**
  - incorporated reliability constraint

- **Requires the evaluation of failure probability P_f for each candidate optimum design considered:**
  - computationally intensive
  - customized approaches to enhance computational efficiency
  (iterative solution techniques, neural network predictions, etc.)

Deterministic design optimization

$$\begin{array}{ll} \text{minimize} & C(\mathbf{d}) \\ \text{subject to} & g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g \\ & d_i \in D, \; i=1,\ldots,n_d \end{array}$$

RBDO

$$\begin{array}{ll} \text{minimize} & C(\mathbf{d}) \\ \text{subject to} & g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g \\ & P_f(\mathbf{d}) \leq P_{f,\max} \\ & d_i \in D, \; i=1,\ldots,n_d \end{array}$$

RRBDO – Reliability and Robustness-Based Design Optimization

- **Treating robustness: a further step beyond controlling reliability**
  - we are interested in reliable and robust structures

- **RBDO already controls reliability**

- **Straightforward optimization approach to treat robustness:**
  built upon RBDO by adding a robustness constraint

RBDO

$$\begin{array}{ll} \text{minimize} & C(\mathbf{d}) \\ \text{subject to} & g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g \\ & P_f(\mathbf{d}) \leq P_{f,\max} \\ & d_i \in D, \; i=1,\ldots,n_d \end{array}$$

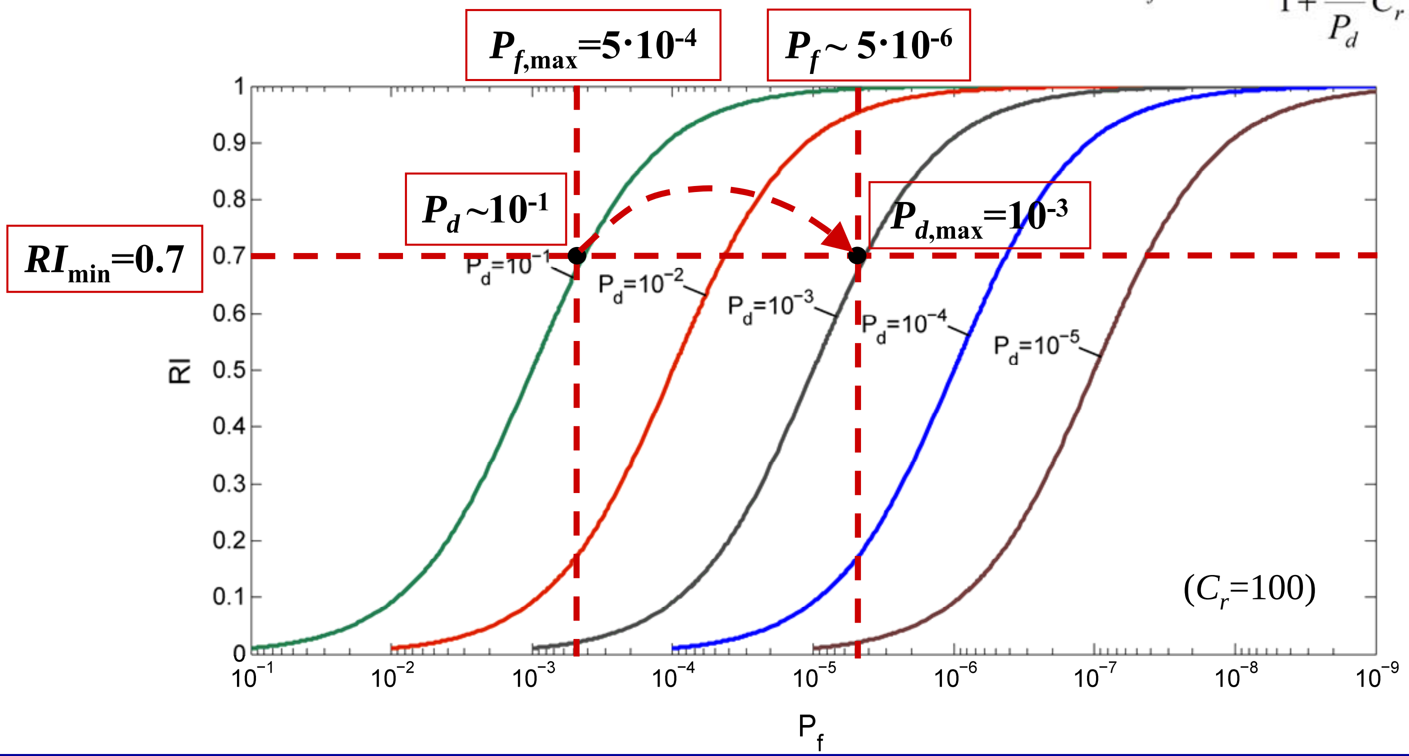
RRBDO

$$\begin{array}{ll} \text{minimize} & C(\mathbf{d}) \\ \text{subject to} & g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g \\ & P_f(\mathbf{d}) \leq P_{f,\max} \\ & RI(\mathbf{d}) \geq RI_{\min} \\ & d_i \in D, \; i=1,\ldots,n_d \end{array}$$

RRBDO – Reliability and Robustness-Based Design Optimization

How are P_f, P_d and RI related?

$$RI = \frac{P_d C_{Dir}}{P_d C_{Dir} + P_f C_{Indir}} = \frac{1}{1 + \frac{P_f}{P_d} C_r}$$



RRBDO – Reliability and Robustness-Based Design Optimization

Thus, the upgrade of RBDO to RRBDO requires additional constraints both on:

- damage probability (P_d)
- robustness (RI)

RBDO

minimize	$C(\mathbf{d})$
subject to	$g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g$
	$P_f(\mathbf{d}) \leq P_{f,\max}$
	$d_i \in D, \; i=1,\ldots,n_d$

RRBDO

minimize	$C(\mathbf{d})$
subject to	$g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g$
	$P_f(\mathbf{d}) \leq P_{f,\max}$
	$P_d(\mathbf{d}) \leq P_{d,\max}$
	$RI(\mathbf{d}) \geq RI_{\min}$
	$d_i \in D, \; i=1,\ldots,n_d$

RRBDO – Reliability and Robustness-Based Design Optimization

RRBDO

minimize	$C(\mathbf{d})$
subject to	$g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g$
	$P_f(\mathbf{d}) \leq P_{f,\max}$
	$P_d(\mathbf{d}) \leq P_{d,\max}$
	$RI(\mathbf{d}) \geq RI_{\min}$
	$d_i \in D, \; i=1,\ldots,n_d$

The constraints on g, P_f, P_d and RI ensure that the final design has:

- **Acceptable system performance** ($P_f(\mathbf{d}) \leq P_{f,\max}$)
- **Acceptable components resistance** ($g_j(\mathbf{d}) \geq 0$, $P_d(\mathbf{d}) \leq P_{d,\max}$)
- **Acceptable confinement of expected direct/ indirect consequences to locally damaged components** ($RI(\mathbf{d}) \geq RI_{\min}$)

RRBDO – Reliability and Robustness-Based Design Optimization

To apply RRBDO, the allowable values $P_{f,\max}$, $P_{d,\max}$, $RI_{\min}$ are required

minimize	$C(\mathbf{d})$
subject to	$g_j(\mathbf{d}) \geq 0, \; j=1,\ldots,n_g$
	$P_f(\mathbf{d}) \leq P_{f,\max}$
	$P_d(\mathbf{d}) \leq P_{d,\max}$
	$RI(\mathbf{d}) \geq RI_{\min}$
	$d_i \in D, \; i=1,\ldots,n_d$

- **$P_{f,\max}$, $P_{d,\max}$ can be taken from codes/guidelines/literature or set according to experience**
- **$RI_{\min}$=?**
  - ***How much is a satisfactory RI?***
  - ***No guidelines/studies/experience yet***
  - ***No universal adoption of a robustness measure***
  - ***No calibration of robustness measure against desired structural performance***

Difficulty in applying RRBDO to practical design cases

MO-RRBDO – Multi-Objective Reliability and Robustness-Based Design Optimization

- **Need for alternative formulation to:**
  - facilitate a more thorough RI-investigation
  - enrich detected design options

- **Upgrade of *RI*:**
  - from being handled in a constraint
  - to being pursued as an objective

RRBDO

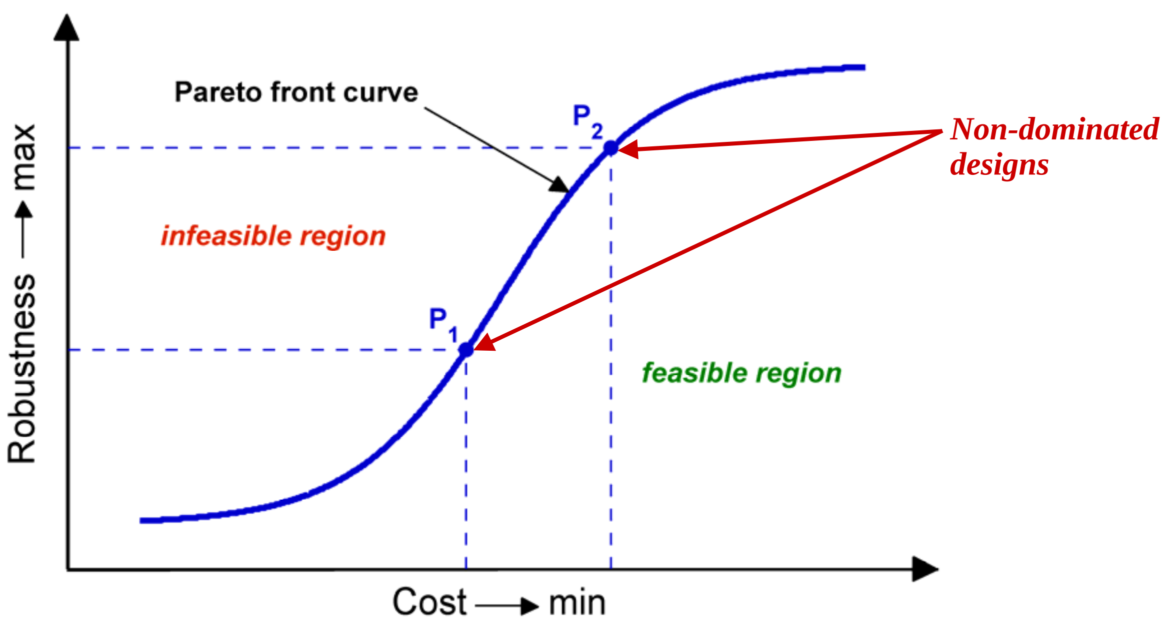
$$
\begin{aligned}
&\text{minimize} && C(\mathbf{d}) \\
&\text{subject to} && g_j(\mathbf{d}) \geq 0,\ j=1,\ldots,n_g \\
& && P_f(\mathbf{d}) \leq P_{f,\max} \\
& && P_d(\mathbf{d}) \leq P_{d,\max} \\
& && RI(\mathbf{d}) \geq RI_{\min} \\
& && d_i \in D,\ i=1,\ldots,n_d
\end{aligned}
$$

MO-RRBDO

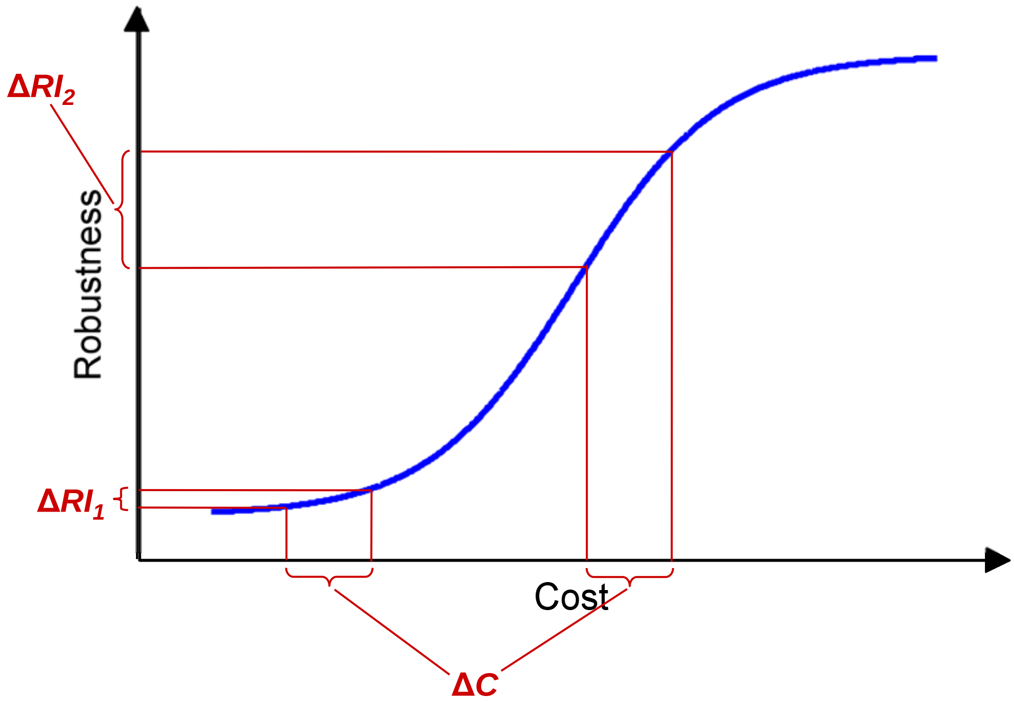
$$
\begin{aligned}
&\text{minimize} && C(\mathbf{d}) \\
&\text{maximize} && RI(\mathbf{d}) \\
&\text{subject to} && g_j(\mathbf{d}) \geq 0,\ j=1,\ldots,n_g \\
& && P_f(\mathbf{d}) \leq P_{f,\max} \\
& && P_d(\mathbf{d}) \leq P_{d,\max} \\
& && d_i \in D,\ i=1,\ldots,n_d
\end{aligned}
$$

MO-RRBDO – Multi-Objective Reliability and Robustness-Based Design Optimization

Typical Pareto front curve obtained by MO-RRBDO

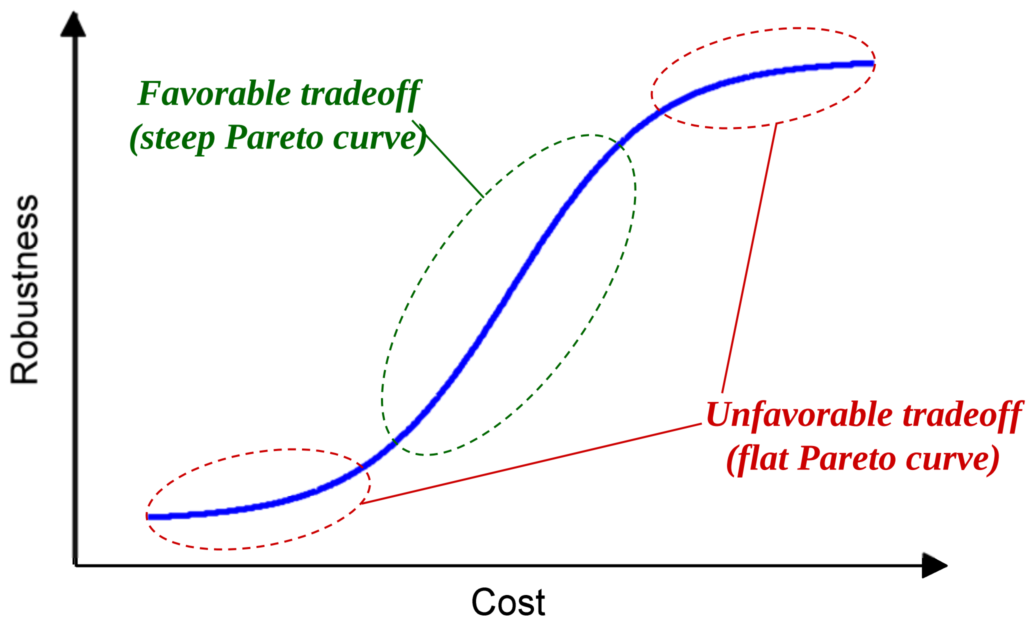


Favorable and unfavorable tradeoff between Cost and Robustness



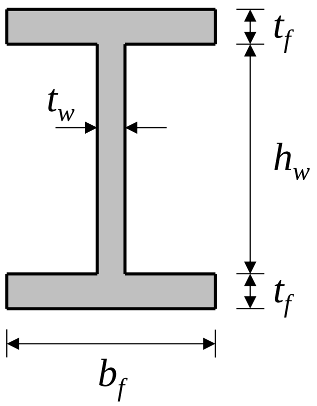
MO-RRBDO – Multi-Objective
Reliability and Robustness-Based Design Optimization

Favorable and unfavorable tradeoff between Cost and Robustness



Numerical example: steel member in pure bending

I-shaped cross-section and 4 design variables



Yield moment: $M_Y = \sigma_Y \dfrac{I}{c}$ → *Performance function monitoring yielding initiation (**damage**)*

$$g_Y = M_Y - M$$

$$P_d = P(g_Y < 0)$$

Plastic moment: $M_P = \sigma_Y A s_1$ → *Performance function monitoring fully plastic deformation (**failure**)*

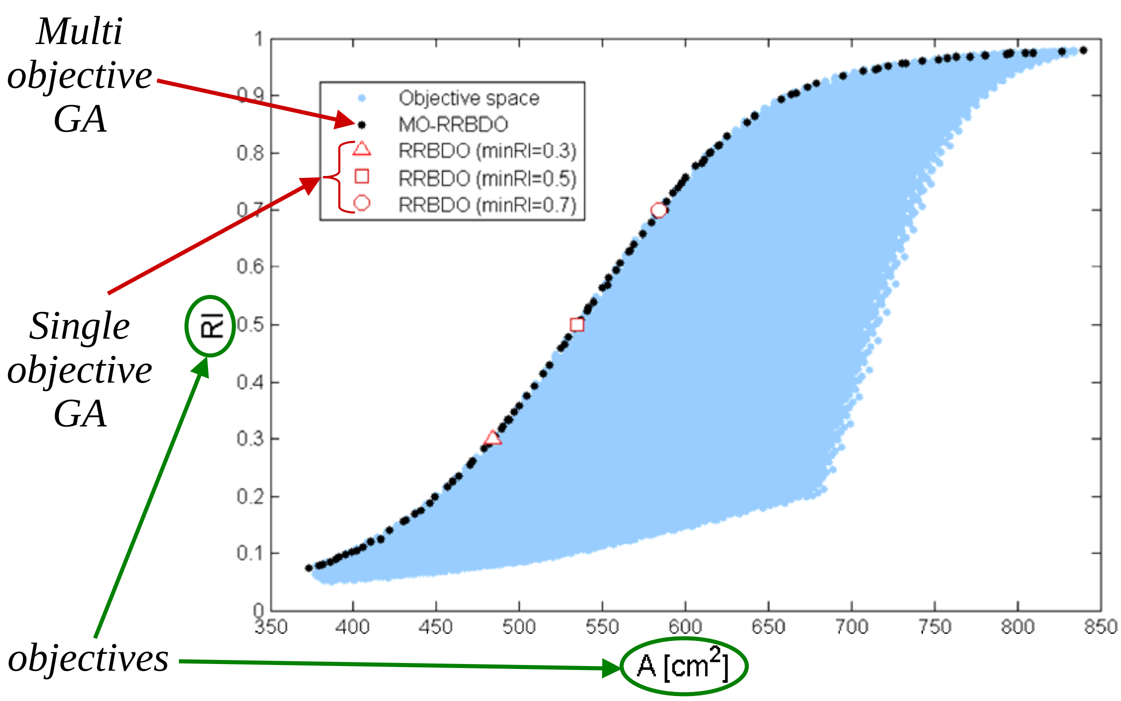
$$g_P = M_P - M$$

$$P_f = P(g_P < 0)$$

Random variable	Probability distribution	Mean value	C.o.V.
Yield stress σ_Y	Normal	250 MPa	7%
Applied moment M	Normal	1500 kNm	25%

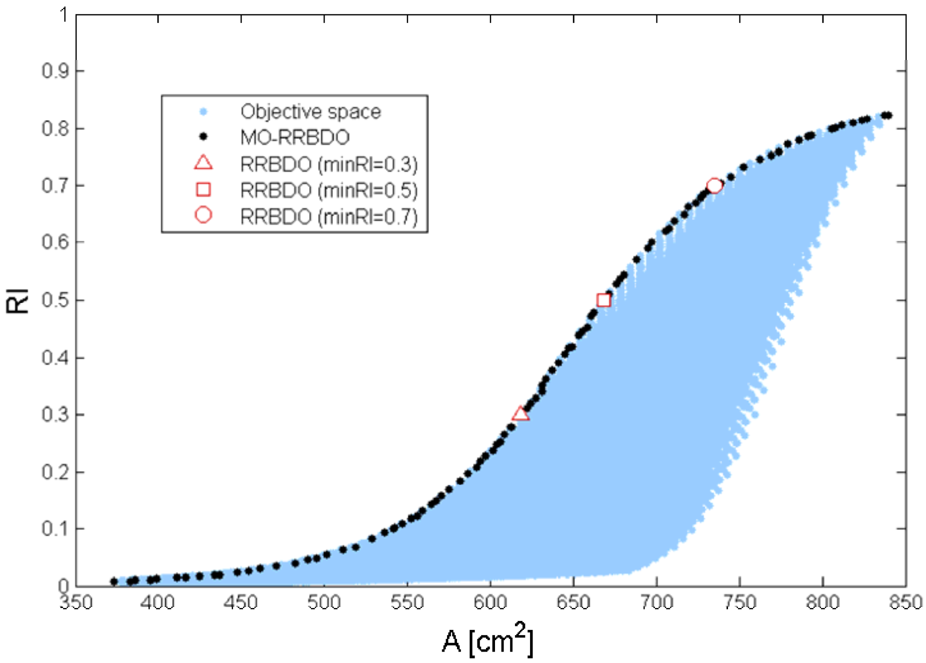
Numerical example: steel member in pure bending

Optimal designs obtained with RRBDO and MO-RRBDO – C_r=100



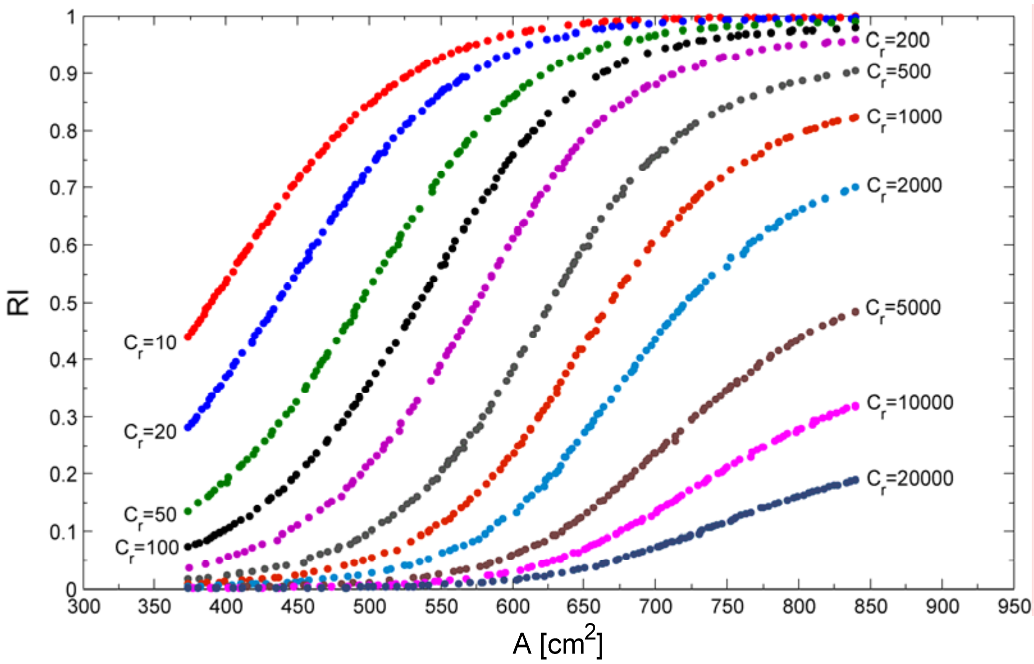
Numerical example: steel member in pure bending

Optimal designs obtained with RRBDO and MO-RRBDO – C_r=1000



Numerical example: steel member in pure bending

Pareto-optimal designs obtained with MO-RRBDO for various C_r-values



RRBDO and MO-RRBDO as tools to assess measures and improvement strategies for robustness

Since structural robustness is a relatively new concept, investigation is required to:

- ***compare alternative robustness measures***
- ***compare simplified robustness measures with 'exact' measure (e.g. for use in codes)***
- ***compare alternative actions to treat robustness***
- ***identify generally applicable and cost-effective actions to improve robustness***

RRBDO and MO-RRBDO as tools to assess measures and improvement strategies for robustness

Aim: perform comparisons – identify suitable actions

Traditional approach

- ***'manual' extensive parametric investigations***
- ***potentially subjective conclusions affected by opinions/preferences/ experience of designer***

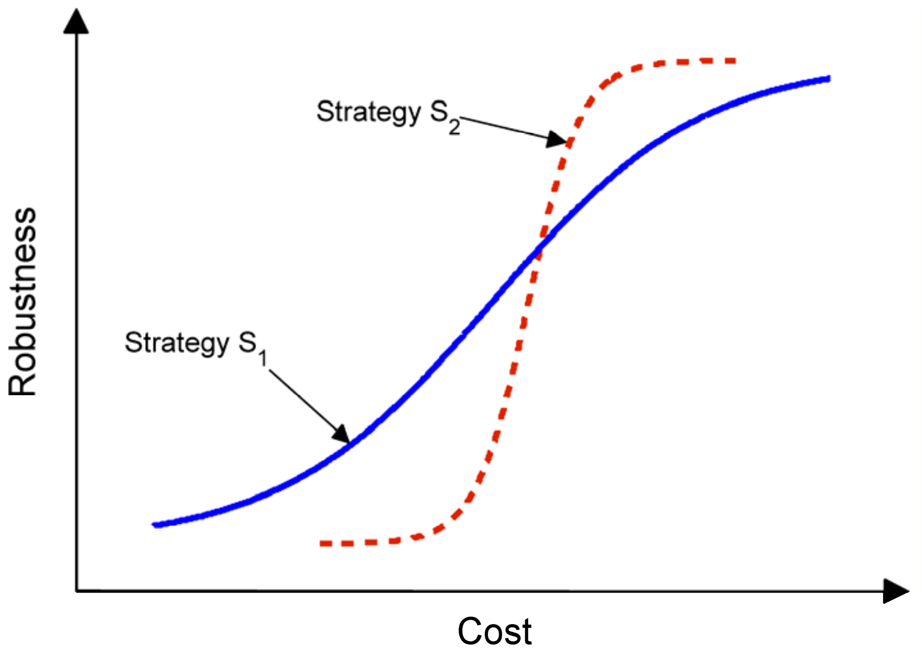
RRBDO / MO-RRBDO approach

- ***automatic extensive investigations***
- ***fair and objective comparisons of competing/controversial actions ⇨ firm/reliable conclusions***
- ***capability to investigate at the edge of design feasibility (limit of satisfaction of constraints)***

of particular interest to COST Action TU0601

RRBDO and MO-RRBDO as tools to assess measures and improvement strategies for robustness

Example: Pareto front curves corresponding to two strategies S_1 and S_2 for improving robustness



Concluding remarks

- **RRBDO and MO-RRBDO: single- and multi-objective design optimization approaches to treat structural robustness**
- **It is envisaged that these new approaches will be exploited to:**
  - detect high-robustness solutions
  - perform tradeoff analysis of competing design objectives
  - perform comparisons
- **Recommendation: use of MO-RRBDO until available information justifies/facilitates the use of RRBDO**
- **Future issue to consider: computational efficiency**
  - the new optimization approaches need to become more tractabe to structural engineering practice

Department of Civil and Environmental Engineering
University of Cyprus
Nicosia – Cyprus

Design optimization methodologies to achieve structural robustness

Dimos C. Charmpis

Robustness of Structures
COST Action TU0601
1st Workshop, February 4-5, 2008, ETH Zurich, Zurich, Switzerland